Section 4.1 Solving Systems of Linear Inequalities

Question 1 – How do you graph a linear inequality?

Question 2 – How do you graph a system of linear inequalities?

Question 1 – How do you graph a linear inequality?

<u>Key Terms</u>

Linear inequality

<u>Summary</u>

A linear inequality is a linear equation where the equal sign has been changed an inequality like $<$, $>$, $\leq$, or $\geq$. The solution set to a linear inequality is a region of the xy plane that is bordered by a line. If the inequality includes an equal sign ($\leq$ or $\geq$), then the border is drawn with a solid line. The solid line indicates that the border is a part of the solution set to the inequality. If the inequality does not include an equal sign ($<$ or $>$), then the border is drawn with a dashed line. The dashed line indicates that the border is not a part of the solution set to the inequality.

To graph the solution set to an inequality,

1. Identify the independent and dependent variables. Begin the graph of the solution set by labeling the independent variable on the horizontal axis and the dependent variable on the vertical axis.
2. Change the inequality to an equation by replacing the inequality with an equal sign.
3. Graph the equation using the intercepts or another convenient method. If the inequality is a strict inequality, like $<$ or $>$, graph the line with a dashed line. If the inequality includes an equal sign, like $<$ or $>$, graph the line as a solid line.
4. Pick a test point to substitute into the inequality. Test points that include zeros are easiest to work with. This test point must not be a point on the line.
5. If substituting the test point into the inequality makes it true, shade the side of the line containing the test point. If substituting the test point into the inequality makes it false, shade the side of the line that does not contain the test point.

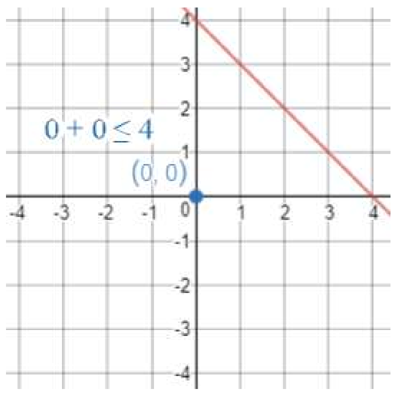
<u>Notes</u>

Guided Example

Graph the inequality on a plane.

$$x + y \leq 4$$

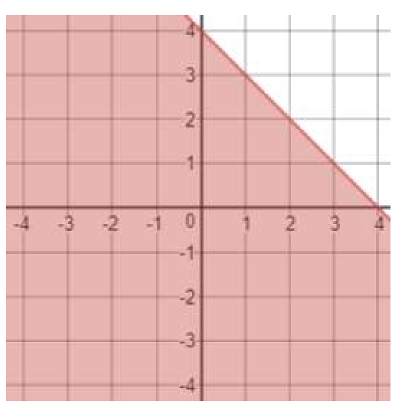
Solution Start by choosing x to be the independent variable and y the dependent variable. The equation of the border of the solution set is $x + y = 4$. We can graph this equation using intercepts or by rewriting the equation as $y = -x + 4$. This results in the graph below.



Since the inequality includes an equal sign, draw the line as a solid line. To see which side of the line to shade, test the point (0,0) in the inequality:

$$0 + 0 \leq 4 \quad \text{TRUE}$$

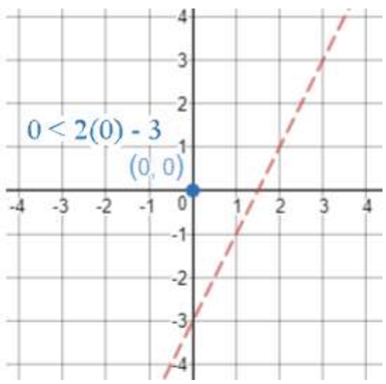
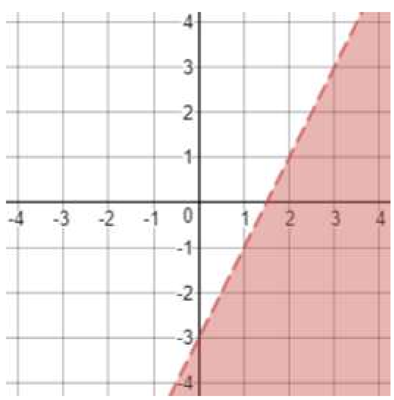
Since the inequality is true, all points on that side of the line satisfy the inequality and the solution set is

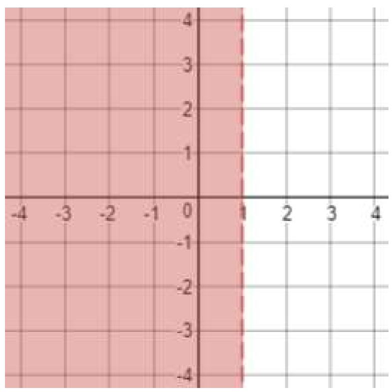


Practice

1. Graph the inequality on a plane.

$$x - y \geq 1$$

Guided Example	Practice
Graph the inequality. $y < 2x - 3$ **Solution** The equation of the border is $y = 2x - 3$. Graph the border with a dashed line and test the point (0, 0) in the inequality.  Since 0 < -3 is false, the solution set is on the other side of the line. 	2. Graph the inequality. $y < -2x + 3$

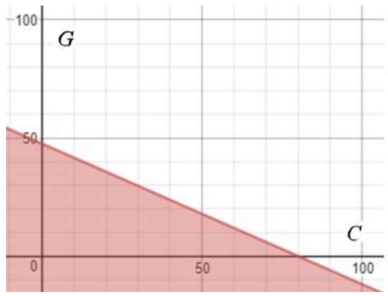
Guided Example	Practice
Graph the inequality on a plane. $x<1$ **Solution** The graph of $x = 1$ is a vertical line. When we test the point (0, 0) in the inequality, we get $0 < 1$ which is true. Shade the side of the line that include this point to get the solution set.  Notice that the border is drawn with a dashed line since the inequality does not include and equals sign.	3. Graph the inequality on a plane. $y > 1$

Guided Example	Practice
Hops costs for a brewer are \$25 per pound for Citra hops and \$42 per pound for Galena hops. How many pounds of each hops should the brewer use if she wants to spend no more than \$2000 on hops? Express your answer as a linear inequality with appropriate nonnegative restrictions and draw its graph. **Solution** Start by defining the variable C for the number of pounds of Citra hops and G for the number of pounds of Galena hops. Either variable may be chosen as the independent variable. For this example, we'll choose C as the independent variable. If Citra hops cost \$25 per pound, then C pounds will cost $25C$. Similarly, G pounds of Galena	4. Ingredient costs for a pet food manufacturer are \$5 per pound for vegetables and \$8 per pound for meat. How many pounds of each ingredient should the manufacturer use if she wants to spend no more than \$4000 on ingredients? Express your answer as a linear inequality with appropriate nonnegative restrictions and draw its graph.

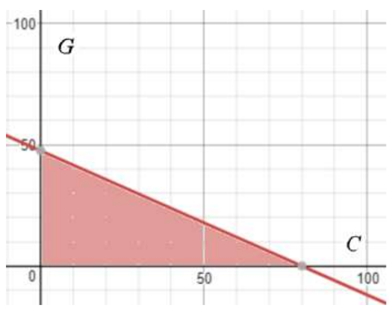
hops will cost $42G$. Since the total cost must be no more than $2000,

$$25C + 42G \le 2000$$

Since (0, 0) makes this inequality true, the solution to this inequality is



However, the variables in this problem represent pounds of hops and can’t be negative. This means the solution must lie only in the first quadrant.



Question 2 – How do you graph a system of linear inequalities?

<u>Key Terms</u>

System of linear inequalities
Bounded region
Unbounded region
Feasible region

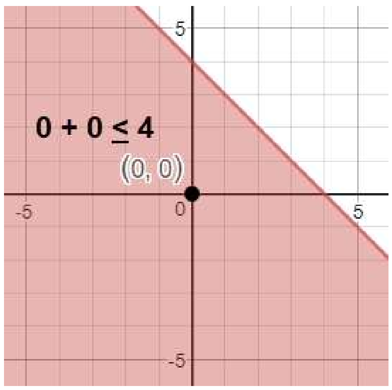
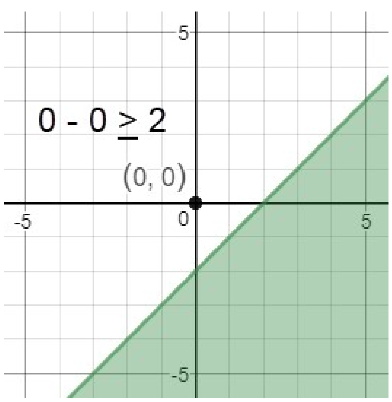
<u>Summary</u>

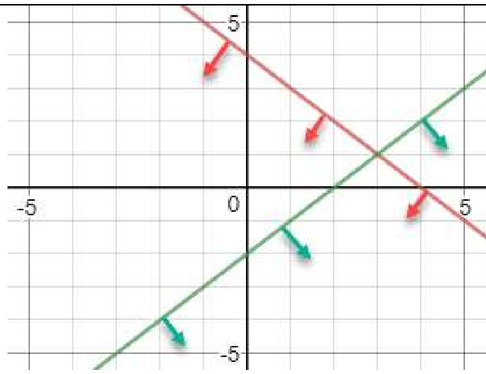
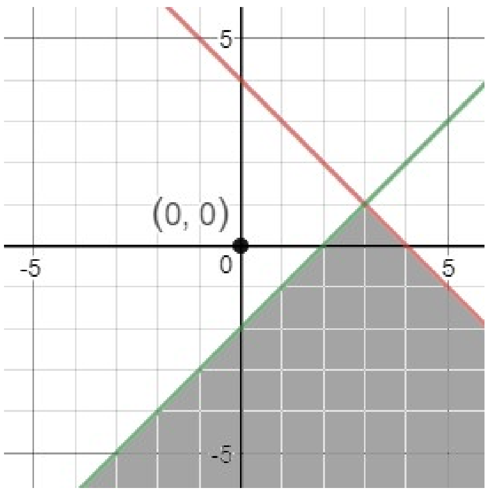
A system of linear inequalities is a group of more than one inequality. To graph a system of linear inequalities,

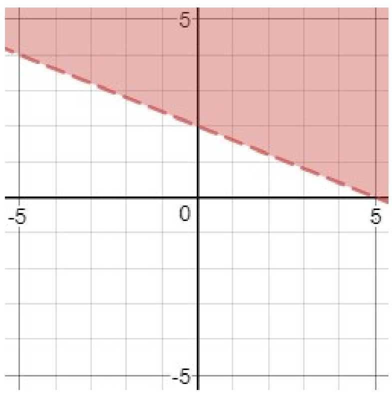
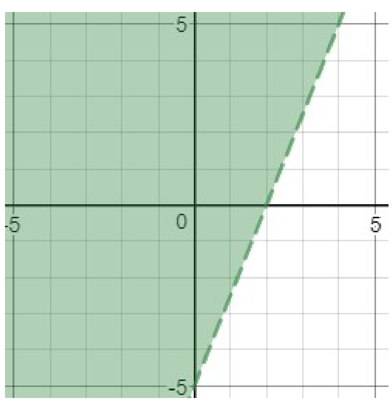
1. Graph the corresponding linear equation for each of the linear inequalities. If the inequality includes an equal sign, graph the equation with a solid line. If the inequality does not include an equals, graph the equation with a dashed line.
2. For each inequality, use a test point to determine which side of the line is in the solution set. Instead of using shading to indicate the solution, use arrows along the line pointing in the direction of the solution.
3. The solution to the system of linear inequalities is all areas on the graph that are in the solution of all of the inequalities. Shade any areas on the graph that the arrows you drew indicate are in common.

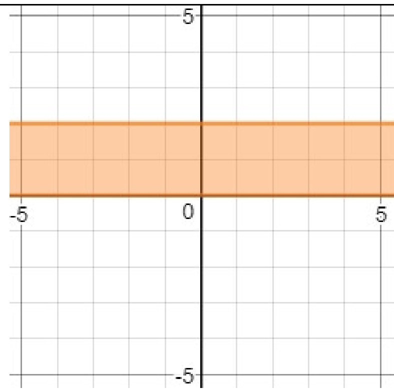
The solution set is often called a feasible region. The feasible region is bounded if it is surrounded by borders on all sides. If the feasible region extends infinitely far in any direction, it is unbounded.

<u>Notes</u>

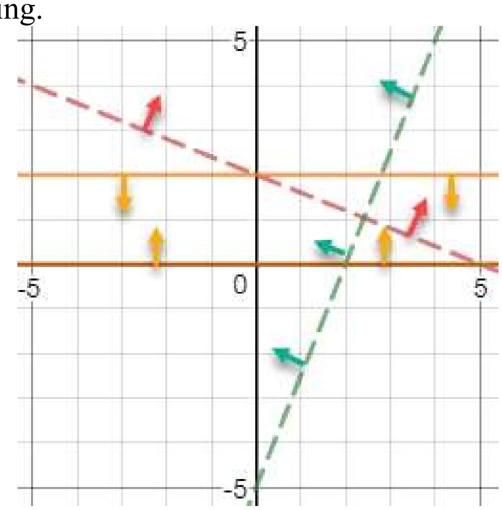
Guided Example	Practice
Graph the feasible region for the following system of inequalities. Tell whether the region is bounded or unbounded. $x+y\le 4$ $x-y\ge 2$ **Solution** Start by graphing the border of the first inequality $x+y=4$. Then test the point (0, 0) in the inequality.  Since the inequality is true, shade the side of the line that (0, 0) lies in. Similarly, shade the line $x-y=6$ and test (0, 0).  Since the inequality is false, shade the opposite side of the line. Using arrows to indicate the shading the region looks like this,	1. Graph the feasible region for the following system of inequalities. Tell whether the region is bounded or unbounded. $x+y\ge 2$ $x-y\le 4$

 These individual solution sets overlap in the solution set of the system of inequalities. 	

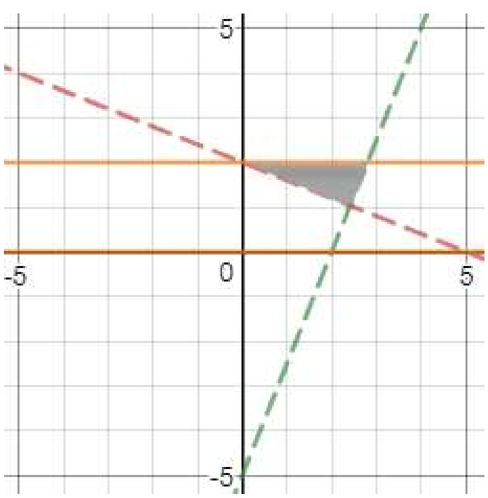
Guided Example	Practice
Graph the solution of the system of linear inequalities. $2x+5y>10$ $5x-2y<10$ $0\le y\le 2$ **Solution** Graph the first inequality $2x+5y>10$.  The second inequality $5x-2y<10$ results in the graph below.  And finally $0\le y\le 2$.	2. Graph the solution of the system of linear inequalities. $6x+3y>18$ $3x-6y<18$ $0\le y\le 1$



Let’s put these together with arrows indicating the shading.



These solution sets overlap in the triangular solution set to the system.

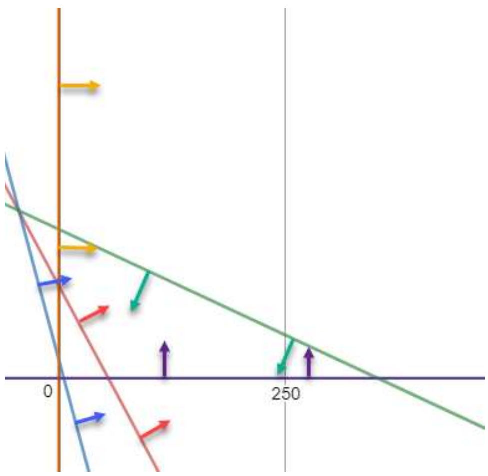


Guided Example	Practice
A pet warehouse is planning to make a package of dog treats containing vegetables and meat. Each ounce of vegetables will supply 1 unit of protein, 2 units of carbohydrates, and 0.25 unit of fat. Each ounce of meat will supply 1 unit of protein, 1 unit of carbohydrates, and 1 unit of fat. Every package must provide at least 55 units of protein, at least 12 units of carbohydrates, and no more than 88 units of fat. Let *x* equal the ounces of vegetable and *y* equal the ounces of meat to be used in each package. a. Write a system of inequalities to express the conditions of the problem. **Solution** The variables are defined in the problem statement. To get started on the inequalities, look for any totals in the problem statement like, "Every package must provide at least 55 units of protein". Since an ounce of vegetables provide 1 unit of protein, *x* ounces of vegetables will provide 1*x* units of vegetables. The protein provided by *y* ounces of meat is 1*y*. This means that we can write a protein constraint $x + y \geq 55$ The constraint for carbohydrates is $2x + y \geq 12$ and the constraint for fat is $0.25x + y \leq 88$ Notice that this inequality is less than or equal to since the problem statement said "no more than 88 units of fat ". In addition to these constraints, we know that each variable should be nonnegative, $x \geq 0 \quad y \geq 0$ Putting these inequalities together gives the system of inequalities,	3. An investor wishes to invest no more than \$10,000 in two stocks. The first stock, Aramid Inc., has a dividend of 1.5%. The second stock, Blue Deuce Insurance, has a dividend of 2%. The investor wishes a total dividend of more than \$144 from these stocks. In addition, the investor wants the amount invested in Aramid to be greater than the amount invested in Blue Deuce. . Let x equal the amount invested in Aramid and y equal the amount invested in Blue Deuce. a. Write a system of inequalities to express the conditions of the problem.

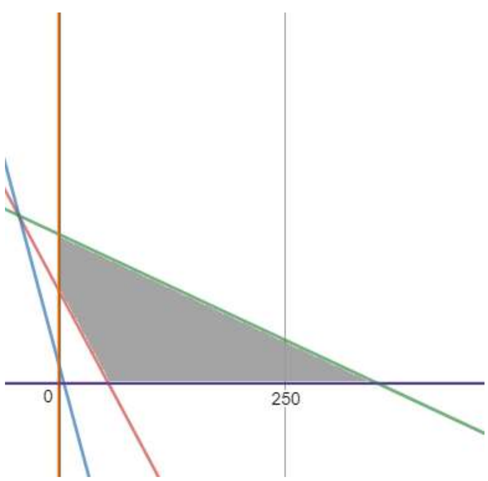
$$x + y \geq 55$$
$$2x + y \geq 12$$
$$0.25x + y \leq 88$$
$$x \geq 0 \quad y \geq 0$$

b. Graph the feasible region of the system.

Solution Draw a line for each border and use arrows to indicate where the solution set for that inequality is. This makes it easier to see where the individual solution sets cross.



The solution to the system of inequalities is the gray feasible region below.



b. Graph the feasible region of the system.