Section 5.1 Simple and Compound Interest

Question 1 – What is simple interest?

Question 2 – What is compound interest?

Question 3 - What is an effective interest rate?

Question 4 - What is continuous compound interest?

Question 1 – What is simple interest?

Key Terms

Future value	Present value
Interest rate	Simple interest

Summary

Simple interest is interest computed on the original principal only. If the present value PV (or principal), in dollars, earns interest at a rate of r for t years, then the interest is

$$I = PV\ rt$$

The future value (also called the accumulated amount or maturity value) is the sum of the principal and the interest. This is the amount the present value grows to after the present value and interest are added.

The future value FV at a simple interest rate r per year is

$$FV = PV + PV\ rt$$

$$= PV\left(1 + rt\right)$$

where PV is the present value that is deposited for t years.

The interest rate r is the decimal form of the interest rate written as a percentage. This means an interest rate of 4% per year is equivalent to $r = 0.04$. Take special care to make sure the time units on the interest rate and time are consistent. If the interest rate is an annual rate, make sure the time is in years. If the interest rate is a monthly rate, make sure the time is in months.

Notes

Guided Example

Find the simple interest on a principal (present value) of $2000 at an annual interest rate of 3% for 8 months.

Solution Simple interest is calculated with $I = PV\,rt$. For this problem, the present value is $PV = 2000$ and the interest rate is $r = 0.03$. Since the interest rate is an annual rate, the time must be in years so $t = \frac{8}{12}$. Put these values into the formula to give

$$I = 2000\left(0.03\right)\left(\tfrac{8}{12}\right) = 40$$

The simple interest is $40.

Practice

1. Find the simple interest on a principal (present value) of $1200 at an annual interest rate of 8% for 10 months.

Guided Example

A loan of $15,500 was repaid at the end of 18 months. If an 6% annual rate of interest was charged, what size repayment check (present value and simple interest) was written?

Solution To find the future value of $15,500, use

$$FV = PV\left(1 + rt\right)$$

with a present value $PV = 15,500$, an interest rate $r = 0.06$ and time $t = \frac{18}{12}$. When these values are substituted, you get

$$FV = 15,500\left(1 + \left(0.06\right)\left(\tfrac{18}{12}\right)\right) = 16,895$$

The repayment check would be written for $16,895.

Practice

2. A loan of $12,700 was repaid at the end of 60 months. If an 9% annual rate of interest was charged, what size repayment check (present value and simple interest) was written?

Guided Example

If $1375 earned simple interest of $502.56 in 86 months, what was the simple interest rate?

Solution Substitute the present value $PV = 1375$, interest $I = 502.56$, and time $t = \frac{86}{12}$ into $I = PV\,rt$ and solve for r:

$$502.56 = 1375 \cdot r \cdot \left(\tfrac{86}{12}\right)$$

$$\frac{502.56}{1375 \cdot \left(\frac{86}{12}\right)} = r$$

$$0.051 \approx r$$

The interest rate is approximately 5.1%.

Practice

3. If $2000 earned simple interest of $345.56 in 90 months, what was the simple interest rate?

Question 2 – What is compound interest?

Key Terms

Compound interest Nominal rate

Interest rate per period Conversion period

Summary

The future value FV of the present value PV compounded over n conversion periods at an interest rate of i per period is

$$FV = PV\left(1+i\right)^n$$

where the interest rate per period is

$$i = \frac{r}{m} = \frac{\text{nominal rate}}{\text{number of conversion periods in a year}},$$

and

$$n = mt = \left(\text{number of conversion periods in a year}\right)\left(\text{term in years}\right).$$

Notes

Guided Example	Practice
Suppose that \$25,000 is invested at 8% interest. Find the amount of money in the account after 4 years if the interest is compounded quarterly. **Solution** Use the compound interest formula, $$FV = PV\left(1+i\right)^n$$ where the present value is $PV = 25,000$, the interest rate per conversion period is $i = \frac{0.08}{4}$ or 0.02, and the number of periods is $n = 4 \cdot 4$ or 16. Using these values, you get $$FV = 25000\left(1+0.02\right)^{16} \approx 34,319.64$$ The account will have approximately \$34,319.64.	1. Suppose that \$30,000 is invested at 7% interest. Find the amount of money in the account after 18 years if the interest is compounded semiannually.

Guided Example

Suppose \$5000 grows to \$8300 in 7 years. What is the annual interest rate if interest is compounded semiannually?

Solution Use the compound interest formula,

$$FV = PV\left(1+i\right)^n$$

where the present value is $PV = 5,000$, the future value is $FV = 8300$, and the number of periods is $n = 7 \cdot 2$ or 14. Put these values into the compound interest formula and solve for *i*:

$$8300 = 5000\left(1+i\right)^{14}$$

$$\frac{8300}{5000} = \left(1+i\right)^{14}$$

$$\sqrt[14]{\frac{8300}{5000}} = \sqrt[14]{\left(1+i\right)^{14}}$$

$$\sqrt[14]{\frac{8300}{5000}} = 1+i$$

$$\sqrt[14]{\frac{8300}{5000}} - 1 = i$$

$$0.03686 \approx i$$

If the rate per conversion period is 0.0369 and there are two conversion periods per year (semiannual), then the nominal rate is $r \approx 0.03686 \cdot 2$ or approximately 7.37%.

Practice

2. Suppose \$10,000 grows to \$15,575 in 5 years. What is the annual interest rate if interest is compounded quarterly?

Guided Example	Practice
Find the present value if the future value is \$14,520.35 and compounded annually at a nominal rate of 1.256% for 6 years. Solution Use the compound interest formula, $$FV = PV\left(1+i\right)^n$$ where the future value is $FV = 14,520.35$, the interest rate per period is $i = 0.01256$ and the number of periods is $n = 6$. Put these values into the compound interest formula and solve for PV: $$14520.35 = PV\left(1+0.01256\right)^6$$ $$\frac{14520.35}{\left(1+0.01256\right)^6} = PV$$ $$13472.63 \approx PV$$ To accumulate \$14520.35, you would need to start with approximately \$13472.63.	3. Find the present value if the future value is \$26,500 and compounded quarterly at a nominal rate of 3.75% for 20 years.

Question 3 – What is an effective interest rate?

Key Terms

Effective interest rate

Summary

The effective interest rate is the simple interest rate that leads to the same future value in one year as the nominal interest rate compounded *m* times per year.

The effective interest rate is

$$r_e = \left(1 + \frac{r}{m}\right)^m - 1$$

where *r* is the nominal interest rate, and *m* is the number of conversion periods per year.

Another name for the effective interest rate is the effective annual rate.

The future value *FV* compounded at an effective interest rate (APY) of r_e is

$$FV = PV\left(1 + r_e\right)^t$$

where *PV* is the present value or principal, and *t* is the term in years.

Notes

Guided Example	Practice
Determine the effective rate for \$1000 invested for 1 year at 7.40% compounded quarterly. **Solution** The effective interest rate r_e is $$r_e = \left(1 + \frac{r}{m}\right)^m - 1$$ where the nominal rate is $r = 0.074$ and the number of compounding periods in a year is $m = 4$. Using these values, you get $$r_e = \left(1 + \frac{0.074}{4}\right)^4 - 1 \approx 0.0761$$ or 7.61%.	1. Determine the effective rate for \$500 invested for 1 year at 2.2% compounded monthly.

Question 4 – What is continuous compound interest?

Key Terms

Continuous interest rate

Summary

The future value *FV* of the present value *PV* compounded continuously at a nominal interest rate of *r* per year is

$$FV = PV\, e^{rt}$$

where *t* is the time in years.

The effective rate r_e for a continuous rate of *r* is

$$r_e = e^r - 1$$

Notes

Guided Example

Find the future value of \$3000 compounded continuously at an annual interest rate of 5.1% for 18 months.

Solution To find the future value for continuous interest, use the formula

$$FV = PV\,e^{rt}$$

with the present value $PV = 3000$, interest rate $r = 0.051$, and time $t = \frac{18}{12}$ or 1.5. Using these values, you get

$$FV = 3000\,e^{(0.051)(1.5)} \approx 3238.51$$

The future value of \$3000 is \$3238.51.

Practice

1. Find the future value of \$5500 compounded continuously at an annual interest rate of 1.1% for 36 months.

Guided Example

How much should be deposited today at a continuous interest rate of 3.5% to accumulate \$10,500 in 5 years?

Solution To find the present value for continuous interest, use the formula

$$FV = PV\,e^{rt}$$

with the future value $FV = 10,500$, interest rate $r = 0.035$, and time $t = 5$. Using these values, solve for the present value PV:

$$10500 = PV\,e^{(0.035)(5)}$$
$$\frac{10500}{e^{(0.035)(5)}} = PV$$
$$8814.30 \approx PV$$

If you start with approximately \$8814.30, it will grow to \$10,500 in 5 years at a continuous interest rate of 3.5% per year.

Practice

2. How much should be deposited today at a continuous interest rate of 5.75% to accumulate \$100,000 in 40 years?