

## Section 5.3 Annuities

Question 1 – What is an ordinary annuity?

Question 2 – What is an annuity due?

Question 3 - What is a sinking fund?

Question 1 – What is an ordinary annuity?

### Key Terms

Ordinary annuity

### Summary

A sequence of payments or withdrawals made to or from an account at regular time intervals is called an annuity. The term of the annuity is length of time over which the payments or withdrawals are made. There are several different types of annuities. An annuity whose term is fixed is called an annuity certain. An annuity that begins at a definite date but extends indefinitely is called a perpetuity. If an annuities term is not fixed, it is called a contingent annuity.

The payments for an annuity may be made at the beginning or end of the payment period. In an ordinary annuity, the payments are made at the end of the payment period. An annuity in which the payment period coincides with the interest conversion period is called a simple annuity.

If equal payments of  $PMT$  are made into an ordinary annuity for  $n$  periods at an interest rate of  $i$  per period, the future value of the annuity  $FV$  is

$$FV = PMT \left[ \frac{(1+i)^n - 1}{i} \right]$$

### Notes

Guided ExamplePractice

An investor deposits \$1500 in a simple annuity at the end of each month. This annuity earns 8% per year, compounded monthly.

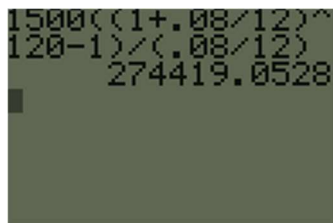
- a. Find the future value if payments are made for ten years.

**Solution** Use the ordinary annuity formula

$$FV = PMT \left[ \frac{(1+i)^n - 1}{i} \right]$$

with payment  $PMT = 1500$ , interest rate per period  $i = \frac{0.08}{12}$ , and the number of periods  $n = 12 \cdot 10$  or 120. With these values, the future value is

$$FV = 1500 \left[ \frac{\left(1 + \frac{0.08}{12}\right)^{120} - 1}{\frac{0.08}{12}} \right] \approx 274,419.05$$



1500((1+.08/12)<sup>120</sup>-1)/(.08/12)  
274419.0528

The total amount of the 120 payments is \$180,000 so the annuity has earned \$274,419.05 - \$180,000 or \$94,419.05 in interest.

- b. What would the investor need to pay each month to accumulate \$500,000 in ten years?

**Solution** In this part, we are given the future value  $FV = 500000$ , interest rate per period  $i = \frac{0.08}{12}$ , and the number of periods  $n = 12 \cdot 10$  or 120. Put these values into the ordinary annuity formula,

$$500000 = PMT \left[ \frac{\left(1 + \frac{0.08}{12}\right)^{120} - 1}{\frac{0.08}{12}} \right]$$

1. An employee deposits \$200 in a simple annuity at the end of each two week pay period. This annuity earns 10% per year, compounded biweekly.

- a. Find the future value if payments are made for thirty years.

- b. What would the employee need to pay each pay period to accumulate \$1,000,000 in thirty years?

To get the payment PMT we need to divide both sides by the quantity in brackets,

$$\frac{500000}{\left[ \frac{\left(1 + \frac{0.08}{12}\right)^{120} - 1}{\frac{0.08}{12}} \right]} = PMT$$

We can do this on the calculator by carrying out the calculation below.

The calculator screen shows the following steps and results:

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((1+.08/12)^120-
1)/(.08/12)
182.9460352
500000/Ans
2733.046384
  
```

Each payment would need to be approximately 2733.05.

### Guided Example

An employee's retirement account currently has a balance of \$100,000. Suppose the employee contributes \$500 at the end of each month. If the account earns a return of 5% compounded monthly, what will the future value of the account in 15 years?

**Solution** To find the future value of this situation, we need to break it into two parts. In the first part, \$100,000 grows with compound interest of 5% compounded monthly for 15 years. In the second part, the employee deposits \$500 each month into an ordinary annuity that earns 5% compounded monthly for 15 years. The future value will be the sum of these pieces,

$$\begin{aligned} FV &= 100000 \left(1 + \frac{0.05}{12}\right)^{180} + 500 \left[ \frac{\left(1 + \frac{0.05}{12}\right)^{180} - 1}{\frac{0.05}{12}} \right] \\ &\approx 211370.39 + 133644.47 \\ &\approx 345014.86 \end{aligned}$$

The future value will be \$345,014.86.

Practice

2. An employee's retirement account currently has a balance of \$50,000. Suppose the employee contributes \$500 at the end of each month. If the account earns a return of 6% compounded monthly, what will the future value of the account in 10 years?

Question 2 – What is an annuity due?

Key Terms

Annuity due

Summary

In an annuity due, payments are made at the beginning of the period instead of the end of the period. If equal payments of PMT are made into an annuity due for  $n$  periods at an interest rate of  $i$  per period, the future value of the annuity  $FV$  is

$$FV = \text{PMT} \left[ \frac{(1+i)^{n+1} - 1}{i} \right] - \text{PMT}$$

For an annuity due, an extra period of interest is earned (the  $n + 1$  in the power) and there is no payment at the end that earns no interest (so PMT is subtracted).

NotesGuided Example

An investor deposits \$500 in a simple annuity at the beginning of each quarter. This annuity earns 2% per year, compounded quarterly. Find the future value if payments are made for five years.

**Solution** Use the annuity due formula,

$$FV = PMT \left[ \frac{(1+i)^{n+1} - 1}{i} \right] - PMT$$

with payment  $PMT = 500$ , interest rate per period  $i = \frac{0.02}{4}$ , and number of periods  $n = 4 \cdot 5$  or 20. Put the values in to give,

$$FV = 500 \left[ \frac{\left(1 + \frac{0.02}{4}\right)^{20+1} - 1}{\frac{0.02}{4}} \right] - 500 \approx 10542.01$$

Since 20 deposits of \$500 each would make the total payments \$10,000, the annuity earns \$10,542.01 - \$10,000 or \$542.01 in interest.

Practice

1. Suppose you deposits \$1000 in a simple annuity at the beginning of every six months. This annuity earns 1% per year, compounded semiannually. Find the future value if payments are made for ten years.

Question 3 – What is a sinking fund?

Key Terms

Sinking fund

Summary

Annuities that are created to fund a purchase at a later date like some equipment or a college education are called sinking funds. In a sinking fund, the future value is known and another quantity in the annuity formula is being solved for.

Notes

Guided ExamplePractice

Suppose you want to accumulate \$2,000,000 in a retirement account in 40 years. The retirement account averages an interest rate of 8% per year. How much would you need to deposit every two weeks (directly from your paycheck) to accumulate \$2,000,000?

**Solution** Since deposits are being made at the end of each two week period, this is an ordinary annuity where the future value is  $FV = 2000000$ , the interest rate per period is  $i = \frac{0.08}{26}$ , and the number of periods is  $n = 26 \cdot 40$  or 1040. Put the values into the ordinary annuity formula,

$$2000000 = PMT \left[ \frac{\left(1 + \frac{0.08}{26}\right)^{1040} - 1}{\frac{0.08}{26}} \right]$$

and solve for the payment  $PMT$ :

$$PMT = \frac{2000000}{\left[ \frac{\left(1 + \frac{0.08}{26}\right)^{1040} - 1}{\frac{0.08}{26}} \right]} \approx 262.85$$

Each payment would need to be approximately \$262.85 to accumulate \$2,000,000.

1. Suppose you want to have \$25,000 in an account in 6 years to purchase a new vehicle. The account earns 3.25% per year. How much would you need to put into the account at the end of each month to accumulate \$25,000?