

Section 6.2 Measuring Central Tendency

Question 1 – What is the difference between a sample and a population?

Question 2 – What is the mean of a dataset?

Question 3 - What is the median of a dataset?

Question 4 - What is the mode of a dataset?

Question 1 – What is the difference between a sample and a population?

Key Terms

Population Sample

Summary

In statistics, we often want to ask questions about very large sets of data. For instance, perhaps we want to know how often a production line produces a defective unit. Over the course of a week, that production line may produce thousands of units. It is not feasible to test every one of the units to determine if it is defective or not.

Instead of looking at every item produced, we might pick a smaller group of units from the production line and determine how many are defective. As long as this group is picked randomly from the larger group of units produced, we can use the information about the smaller group to infer how many defective. The smaller group is called a sample which is selected from the larger group. The larger group is called the population.

Notes

Question 2 – What is the mean of a dataset?

Key Terms

Mean

Summary

The mean of a set of data is calculated by adding up all of the data and then dividing by the total number of data. A population mean is calculated from a large set of data and is denoted by the letter μ .

Population Mean

Let x_i denote the i^{th} observation of a variable x from a population with N total observations. The population mean is

$$\mu = \frac{x_1 + x_2 + \cdots + x_N}{N}$$

A sample mean is denoted by the symbol \bar{x} and is calculated the same way. However, in the case of the sample mean the data is a sample from a larger population.

Sample Mean


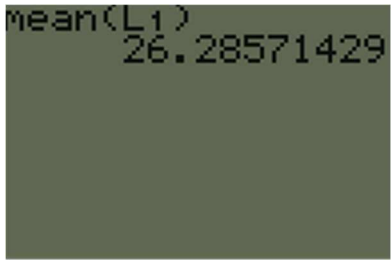
Let x_i denote the i^{th} observation of a variable x from a sample with n total observations. The sample mean is

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

A graphing calculator may be used to calculate the value of the mean for a set of data.

1. Start by entering the data into a list on the calculator. Press **[STAT][ENTER]** to enter a list.
2. Enter each data value under L1 followed by **[ENTER]**.
3. Once each data value has been entered, press **[2nd][MODE]** to quit the list editor and return to the home screen.



<ol style="list-style-type: none">From the home screen, press $\boxed{2nd}\boxed{STAT}$ to access the LIST commands.Press $\boxed{\rightarrow}\boxed{\rightarrow}$ to move the cursor to the MATH menu.Press $\boxed{\downarrow}\boxed{\downarrow}\boxed{ENTER}$ or $\boxed{3}$ to paste the mean(command to the home screen.	
<ol style="list-style-type: none">After the mean(command, we need to insert the name of the list we are using. Press $\boxed{2nd}\boxed{1}\boxed{)}$ to paste the name of the list, L1 and a parentheses into the homescreen.Press \boxed{ENTER} to calculate the mean.	

Notes

Guided Example

A bank measures the amount of time it takes 40 randomly selected customers to make a deposit. These times are recorded in the table below.

2	2	3	1	2	2	2	1	2	2
2	1	2	2	2	5	4	3	2	3
3	4	3	3	2	3	1	3	2	2
2	3	1	2	1	2	2	5	1	2

Find the mean amount of time it takes a customer to make a deposit.

Solution Since this is a sample from a much larger set of data (all customers making deposits), we use the formula

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

to calculate the mean. The sum of the data is 92 and the total number of data values is 40,

$$\bar{x} = \frac{92}{40} = 2.3$$

The mean time it takes a customer to make a deposit is 2.3 minutes.

Practice

1. A manufacturing company measures the number of defective units its production facility produces in an hour. Over the course of several days, it records the data below.

0	3	3	3	0
1	2	3	1	1
0	1	1	2	2
0	0	1	1	1

Find the mean number of defective units produced in an hour.

Guided Example

Suppose the grades on an exam for a class are recorded in the table below.

Class	Frequency
[40, 50)	1
[50, 60)	0
[60, 70)	5
[70, 80)	5
[80, 90)	9
[90, 100)	27

Estimate the mean score on the exam using the midpoint of each class to represent the class.

Solution In this example, the data are given in a frequency table. For instance, the third row of the table indicates that there are 5 data values in the interval [60, 70). We do not know exactly what the data values are so we will estimate all data values in [60, 70) with the representative value 65.

We could include five 65's in the sum for the mean, however, it is easier to simply write $5 \cdot 65$ instead of $65 + 65 + 65 + 65 + 65$. Doing this for each data value gives the sample mean

$$\bar{x} = \frac{1 \cdot 45 + 0 \cdot 55 + 5 \cdot 65 + 5 \cdot 75 + 9 \cdot 85 + 27 \cdot 95}{47} \approx 86.7$$

On a graphing calculator, we can compute the mean of frequency data by entering the data values in L1 and the corresponding frequencies in L2:

L1	L2	L3	Z
45	1		
55			
65	5		
75	5		
85	9		
95	27		

L2(?) =			

Then compute the mean using the mean command followed by the names of the two lists.

mean(L1,L2)
86.70212766

Practice

2. Suppose the grades on an exam for a class are recorded in the table below.

Class	Frequency
[25, 40)	2
[40, 55)	0
[55, 70)	3
[70, 85)	7
[85, 100)	8

Estimate the mean score on the exam using the midpoint of each class to represent the class.

Question 3 – What is the median of a dataset?

Key Terms

Median

Summary

If the data is arranged in numerical order, the median is the center value that splits the data into two halves.



Once the data is arranged in numerical order, the center value may be determined by dividing the total number of data by 2. If the result of dividing the total number of data by 2 is not an integer, round the quotient up to the nearest integer. The center value is located at this position when the data is listed in numerical order.

Let's see how this is done with the data 1, 2, 4, 6, 10. Since there are 5 data values, the median is in position 3 of the list (round up $\frac{5}{2}$ to 3). Since the number in position 3 is 4, the median is 4.

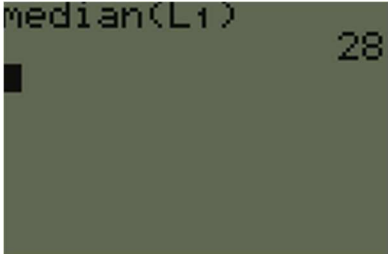
Remember to order the list in numerical order first.

If the quotient is an integer, the median is the mean of the data located in that position and the following position. For instance, the median of the data 2, 4, 7, 12, 45, 45 is the mean of the data located in position 3 and 4. Since those data are 7 and 12, the median is $\frac{7+12}{2}$ or 9.5.

To calculate the median of a set of data, follow the steps below.

<ol style="list-style-type: none"> 1. Start by entering the data into a list on the calculator. Press [STAT][ENTER] to enter a list. 2. Enter each data value under L1 followed by [ENTER]. 3. Once each data value has been entered, press [2nd][MODE] to quit the list editor and return to the home screen. 	 <p>The image shows a TI-84 calculator screen in the list editor. The list L1 contains the values 33, 19, 33, 17, 30, and 28. The cursor is positioned at the end of the list. The screen also shows L2 and L3 are empty, and the prompt 'L1(B)=' is visible at the bottom.</p>
<ol style="list-style-type: none"> 4. From the home screen, press [2nd][STAT] to access the LIST commands. 5. Press [▶][▶] to move the cursor to the MATH menu. 6. Press [▼][▼][▼][ENTER] or [4] to paste the median(command to the home screen. 	 <p>The image shows the MATH menu on a TI-84 calculator. The menu items are: 1:min(, 2:max(, 3:mean(, 4:median(, 5:sum(, 6:Prod(, and 7:stdDev(. The 'median(' option is highlighted with a cursor.</p>

7. After the median(command, we need to insert the name of the list we are using. Press $\boxed{2\text{nd}}\boxed{1}\boxed{)}$ to paste the name of the list, L1 and a parentheses into the homescreen.
8. Press $\boxed{\text{ENTER}}$ to calculate the median.

A photograph of a TI-84 Plus calculator screen. The screen is dark with light-colored text. The text on the screen reads "median(L1)" on the top line and "28" on the second line. A small black cursor is visible on the left side of the screen, positioned between the two lines of text.

Notes

Guided Example

Suppose the grades on an exam for a class are recorded in the table below.

44%	65%	70%	94%	96%	90%	94%	99%	85%	73%
96%	98%	96%	96%	95%	73%	80%	94%	85%	93%
96%	73%	94%	67%	61%	93%	97%	80%	99%	
81%	61%	87%	83%	96%	95%	84%	69%	92%	
91%	90%	78%	88%	97%	90%	97%	91%	97%	

Find the median score for the exam.

Solution To find the median of this set of data, we need to put the data in numerical order. Once this is done, divide the total number of data, 47, by 2. This gives us 23.5 which we round up to 24. The median is the data value that is located in the 24th position of the list.

A graphing calculator may be used to put the list in ascending order. Start by entering the data into list L1.

A calculator screen showing list L1 with the following values: 88, 97, 90, 97, 91, 97. The screen also shows 'L1(48) =' at the bottom.

L1	L2	L3	1
88			
97			
90			
97			
91			
97			
L1(48) =			

Press $\boxed{2\text{nd}}\boxed{\text{MODE}}$ to quit the list editor and return to the homescreen. Press $\boxed{2\text{nd}}\boxed{\text{STAT}}\boxed{\blacktriangleright}$ to access the LIST OPS menu and $\boxed{\text{ENTER}}$ or $\boxed{1}$ to paste SortA(to the homescreen. Enter the name of the list by pressing $\boxed{2\text{nd}}\boxed{1}\boxed{\square}$. Press $\boxed{\text{ENTER}}$ to carry out the command. Now when you access the list by pressing $\boxed{\text{STAT}}\boxed{\text{ENTER}}$, the list will be shown in ascending order.

A calculator screen showing list L1 with the following values: 61, 61, 65, 67, 69, 70. The screen also shows 'L1(1)=44' at the bottom.

L1	L2	L3	1
61			
61			
65			
67			
69			
70			
L1(1)=44			

Use the arrow buttons to scroll to the 24th entry.

L1	L2	L3	1
85			
87			
88			
90			
90			
90			
91			
L1(24) = 91			

The median is 91.

Practice

- Suppose the grades on an exam for a class are recorded in the table below.

35%	38%	63%	92%
76%	55%	65%	77%
85%	98%	71%	75%
88%	89%	91%	75%
71%	79%	88%	85%

Find the median score for the exam.

Question 4 – What is the mode of a dataset?

Key Terms

Mode

Summary

The mode of a dataset is the value that occurs most frequently in the data. To determine how frequently each data value occurs, list them in ascending order to make it easier to count the frequency of each data value.

Notes

Guided Example

Suppose the grades on an exam for a class are recorded in the table below.

44%	65%	70%	94%	96%	90%	94%	99%	85%	73%
96%	98%	96%	96%	95%	73%	80%	94%	85%	93%
96%	73%	94%	67%	61%	93%	97%	80%	99%	
81%	61%	87%	83%	96%	95%	84%	69%	92%	
91%	90%	78%	88%	97%	90%	97%	91%	97%	

Find the mode for the exam.

Solution Examine the data carefully to determine which data value occurs most frequently. The data value 96 occurs six times in the list. Since this is the most frequently occurring data value, the mode is 96.

Practice

1. Suppose the grades on an exam for a class are recorded in the table below.

35%	38%	63%	92%
76%	55%	65%	77%
85%	98%	71%	75%
88%	89%	91%	75%
71%	79%	88%	85%

Find the mode for the exam.