# 8.2 Combinations

## Question 1: What is a combination?

## Question 2: What is the difference between a permutation and combination?

## Question 1: What is a combination?

Key Terms

Combinations

Summary

In Section 8.1, we used permutations to count different arrangements of objects. The word "arrangements" is used since different orders of objects must be counted separately. Different arrangements of numbers and letters on an auto license plate lead to different license plate numbers.

In many applications, different arrangements of objects are not counted differently. Many states have lotteries that are used to fund schools and environmental causes. In these lotteries, lottery officials select numbered balls from a group of balls labeled 1 through a larger number like 52. A player wins the lottery jackpot if the numbers the player selects matches the numbers on the balls selected by officials. The order in which the balls are drawn does not have to be duplicated. The number of ways the balls can be selected from a larger group of balls is calculated using **combinations**. In combinations, groupings of objects are counted and the order of the objects in the grouping are irrelevant.

**Combinations**

A combination of $n$ objects selected $r$ at a time is a group of $r$ objects selected from $n$ different objects without regard to order. The number of combinations is

$$C(n,r) = \frac{n!}{(n-r)!\,r!}$$

For the same values of $n$ and $r$, the number of combinations is always smaller than the number of permutations due to the $r!$ factor in the denominator. This makes sense since combinations do not count the rearrangements of the letters, only the different groupings.

For example, in the last section we looked at the permutations of the letters CAT taken two at a time:

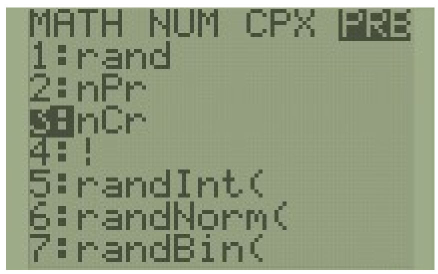
| | | |
|---|---|---|
| CA | AT | CT |
| AC | TA | TC |

Since AC and CA (as well as AT and TA, CT and TC) are counted separately, the order of the letters makes a difference. In combinations, AC and CA are counted as one since order does not make a difference. So, the combinations of the letters CAT taken two at a time is

| | | |
|---|---|---|
| CA | AT | CT |

The number of combinations would be calculated as $C(3,2)=\frac{3!}{(3-2)\,!2!}=3$.

Combination may be computed on a TI graphing calculator using the MATH button and then selecting the PRB submenu.




The third selection will compute a combination when the number of objects is entered in front of nCr and the number of objects selected is entered after the command.


<u>Notes</u>

<u>Guided Example 1</u>

Evaluate the expressions below.

a. $C(7,5)$

**Solution**

$$C(7,5)=\frac{7!}{(7-5)!\,5!}=21$$


b. $C(10,9)$

**Solution**

$$C(10,9)=\frac{10!}{(10-9)!\,1!}=10$$


<u>Practice</u>

Evaluate the expressions below.

a. $C(6,3)$

b. $C(8,7)$

Guided Example 2

A small school has 4 teachers and 50 students.

a. How many different committees can be formed from teachers and students if the committee consists of five people?

**Solution** When counting committees where there are no assigned duties, rearranging committee members yields the same committee. Combinations are used to count the number of committees. Since there is a total of 54 teachers and students, the number of ways to select 5 from the total of 54 is

$$C(54,5)=\frac{54!}{(54-5)!5!}=3{,}162{,}510$$

This counts the number of ways to grab groups of 5 from a total of 54 people where the order of the people in the group is irrelevant.

b. How many different committees can be formed from the students if the committee must have five students?

**Solution** There are 50 students and we wish to find how many ways there is to select groups of 5. Ordering is irrelevant so we calculate with combinations,

$$C(50,5)=\frac{50!}{(50-5)!5!}=2{,}118{,}760$$

c. How many committees of students and teachers may be formed if the committee must have 2 teachers and 3 students?

**Solution** Start by breaking this down into two tasks: count the number of groupings of two teachers from the total of 4 and count the number of groupings of 3 students from the total of 50. Since order does not matter, we can compute each of these with combinations:

Practice

From a group of 10 smokers and 20 nonsmokers, a researcher wants to randomly select smokers and nonsmokers for a study.

a. How many ways is there to select 5 smokers for the study?

b. How many ways is there to select 5 nonsmokers for the study?

c. How many ways is there to select 5 smokers and 5 nonsmokers for the study?

| | |
|---|---|
| $C(4,2)=\frac{4!}{(4-2)!2!}=6$<br>$C(50,3)=\frac{50!}{(50-3)!3!}=19,600$<br><br>Because of the Multiplication Principle, the total number of ways to order teachers and students for the committee is the product of these numbers,<br><br>$C(4,2)\cdot C(50,3)=6\cdot 19,600=117,600$ | |

Guided Example 3 | Practice

| | |
|---|---|
| A state lottery game requires that you pick 5 different numbers from 1 to 54.<br><br>a. If you pick of the numbers correctly, you win \$250,000. In how many ways can you pick the winning numbers without regard to order?<br><br>**Solution** The order in which lottery numbers are picked does not matter. To win the prize, you must match five numbers out of five numbers without regard to order. The number of ways to do this is<br><br>$C(5,5)=\frac{5!}{(5-5)!5!}=1$<br><br>b. A lesser prize is won if you select 4 of the winning numbers out of 5. In how many ways can you pick lesser winning numbers without regard to order?<br><br>**Solution** You need to pick 4 of the winning numbers along with a non-winning number. The number of ways to pick the winning numbers is<br><br>$C(5,4)=\frac{5!}{(5-4)!4!}=5$<br><br>If 5 numbers are winning numbers, the other 49 must be losing numbers. The number of ways to pick the lesser prize is | A state lottery game requires that you pick 4 different numbers from 1 to 36.<br><br>a. If you pick of the numbers correctly, you win \$50,000. In how many ways can you pick exactly of the winning numbers without regard to order?<br><br>b. A lesser prize is won if you select 3 of the winning numbers out of 4. In how many ways can you pick lesser winning numbers without regard to order? |

| | |
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| $$C(5,4)\cdot 49 = 5\cdot 49 = 245$$ c. How many ways are there to pick 5 different lottery numbers from 1 to 54 without regard to order? **Solution** From the 54 numbers, we want to pick a group of 5. Using combinations, we get $$C(54,5)=\frac{54!}{(54-5)!5!}=3,162,510$$ | c. How many ways are there to pick 4 different lottery numbers from 1 to 36 without regard to order? |

# Question 2: What is the difference between a permutation and combination?

Key Terms

Summary

We have looked at several ways of counting. Each counting strategy is applicable under certain assumptions. The table below outlines the assumptions and formulas for each strategy.

| Strategy | Multiplication Principle | Permutations | Combinations |
|---|---|---|---|
| Purpose | Number of ways to make $n$ choices where there are $d_i$ ways to make $i^{th}$ choice | Number of ways to select $r$ objects from $n$ different objects | |
| Repetition Allowed? | Yes | No | No |
| Order Important? | Yes | Yes | No |
| Formula | $d_1 \cdot d_2 \cdot \ldots \cdot d_n$ | $P(n,r) = \frac{n!}{(n-r)!}$ | $C(n,r) = \frac{n!}{(n-r)!\,r!}$ |

Sone problems may require more than one strategy. In those situations, it helps to break the problem into several choices, count the number of ways the choices may be made, and then multiply the choices using the Multiplication Principle.

Notes

<table>
<tr><th><u>Guided Example 4</u></th><th><u>Practice</u></th></tr>
<tr><td>A group of 15 workers decides to send a delegation of 4 to their supervisor to discuss their work assignments. In the following cases, determine the possible number of delegations.<br><br>a. The number of possible 4 person delegations.<br><br>Solution In the delegation sent to the supervisor, there are no assigned roles so the ordering are irrelevant, In addition, each person may only ne sent once so repetition is not allowed. Using combinations, the number of possible delegations is<br><br>$$C(15,4)=\frac{15!}{(15-4)!4!}=1365$$<br><br>b. The number of 4 person delegations that include the foreman.<br><br>Solution Since the foreman is a required member of the delegation, only three remaining members will be selected from the 14 remaining workers. The number of possible delegations is<br><br>$$C(14,3)=\frac{14!}{(14-3)!3!}=364$$<br><br>c. Suppose there are 7 women and 8 men in the group. How many delegations include exactly one woman?<br><br>Solution Since repetition is not allowed and order makes no difference, we will still use combinations to count the delegations. However, we need to break the delegation into two decisions and apply the Multiplication Principle:<br><br>$$\underset{\text{choose woman}}{\underline{C(7,1)}}\cdot\underset{\text{choose men}}{\underline{C(8,4)}}=7\cdot 70=490$$</td><td>A town is forming a committee to investigate ways to improve water conservation in the town.<br><br>a. In how many ways can a committee of 8 be selected from a group of 29 people?<br><br>b. To make sure all constituencies are represented, the committee will consist of 2 members from the 7-person town council, 3 members of a 10-person citizens advisory board, and 3 members of the town's 12-person utility department. How many ways can that committee be formed?</td></tr>
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Guided Example 5

Licenses plates on automobiles consists of letters and numbers where the letters and numbers may be repeated.

a. How many licenses plates are there with three letters followed by three numbers?

**Solution** In this situation, different arrangements of letters yield different license plates, so order makes a difference. Additionally, the letters and numbers may be repeated. Because of these facts, we'll apply the Multiplication Principle where each decision regards a letter or number:

$$\underset{\text{first letter}}{\underline{26}} \cdot \underset{\text{second letter}}{\underline{26}} \cdot \underset{\text{third letter}}{\underline{26}} \cdot \underset{\text{first number}}{\underline{10}} \cdot \underset{\text{second number}}{\underline{10}} \cdot \underset{\text{third number}}{\underline{10}} = 17,576,000$$

There are a total of 17,576,000 license plates with three letters followed by three numbers.

b. How many license plates are the with six letters or numbers?

**Solution** In this case, any of the entries on the license plate may be a letter of number. Applying the Multiplication Principle yields

$$\underset{\text{first entry}}{\underline{36}} \cdot \underset{\text{second entry}}{\underline{36}} \cdot \underset{\text{third entry}}{\underline{36}} \cdot \underset{\text{fourth entry}}{\underline{36}} \cdot \underset{\text{fifth entry}}{\underline{36}} \cdot \underset{\text{sixth entry}}{\underline{36}} = 2,176,782,336$$

a. How many different three-digit numbers can be formed using the digits 1, 3, 5, 7, and 9 without repetition? For example, 911 is not allowed.

b. How many different two-digit numbers can be formed using the digits 2, 4, 5, 6, and 1 with repetition? For example, 11 is allowed.

Guided Example 6

| | |
|---|---|
| An engineering firm with 15 employees is sending a five-person team to deliver a proposal. The team must have a team leader and a main presenter; the other members have no particularly defined roles. In how many ways can this team be formed?<br><br>**Solution** Break the team into two parts: the two people with defined roles and the three people without defined roles.<br><br>Rearranging the three roles would lead to different teams so order makes a difference for that part. Assuming no member can have more than one role, the number of ways to assign the team leader, main presenter is<br><br>$$P(15,2)=\frac{15!}{(15-2)!}=210$$<br><br>Since the remaining member have no defined roles, order does not matter and combinations may be used to compute the number of ways to select three members from the remaining 13 employees,<br><br>$$C(13,3)=\frac{13!}{(13-3)!3!}=286$$<br><br>Because of the Multiplication Principle, these numbers multiply, $210\cdot 286$, to give 60,060 possible teams.<br><br>Note that this problem could also be solve by selecting the members with no defined roles followed tb the team leader and presenter,<br><br>$$C(15,3)\cdot P(12,2)=455\cdot 132=60,060$$ | The PTK Honors program has 20 members. The program's board has a president, vice president, treasurer and two members at large (all of whom must be different).<br><br>a. In how many ways can the board be selected?<br><br>b. The program wants to select two board members to manage a food bank. In how many ways can these board members be selected? |