8.3 Probability with Permutations and Combinations

Question 1: How do you find the likelihood of a certain type of license plate?

Question 2: How do you find the likelihood of a particular committee?

Question 3: How do you find the probability of winning a lottery?

Question 4: How do you find the likelihood of detecting a defective product?

Question 1: How do you find the likelihood of a certain type of license plate?

Key Terms

Summary

To find the likelihood of an event E with equally likely outcomes, we need to count the number of outcomes in the event and divide it by the total number of outcomes in the sample space.

$$P(E) = \frac{n(E)}{n(S)}$$

For this question, the events involve counting license plates. Since the order of the letters and numbers on a license plate matters, we typically use the Multiplication Principle to calculate the number of outcomes in the event and sample space. However, if repetition of letters or numbers are not allowed, we can also use permutations.

Notes

Guided Example 1	Practice
If the license plates in a particular state consist of three letters followed by three numbers, what is the probability that a randomly generated plate begins with the letters "NUT"? **Solution** Since the first three letters must be "NUT", the only choices to made are the numbers. Using the Multiplication Principle yields, $\underset{\text{first number}}{\underline{10}} \cdot \underset{\text{second number}}{\underline{10}} \cdot \underset{\text{third number}}{\underline{10}} = 1000$ To find the total number of license plates that have three letters followed by three numbers, apply the Multiplication Principle again: $\underset{\text{first letter}}{\underline{26}} \cdot \underset{\text{second letter}}{\underline{26}} \cdot \underset{\text{third letter}}{\underline{26}} \cdot \underset{\text{first number}}{\underline{10}} \cdot \underset{\text{second number}}{\underline{10}} \cdot \underset{\text{third number}}{\underline{10}} = 17,576,000$ Dividing these numbers gives thee likelihood that a randomly generated license plate would start with "NUT", $P(\text{starts with NUT}) = \dfrac{n(\text{starts with nut})}{n(\text{3 letters, 3 numbers})} = \dfrac{1000}{17,576,000} \approx 0.000057$ Or approximately 0.0057%.	If the license plates in a particular state consist of four letters followed by three numbers, what is the probability that a randomly generated plate ends with the numbers "666"?

Question 2: How do you find the likelihood of a particular committee?

Key Terms

Summary

As we saw earlier, committees with no defined roles are counted with combinations since order does not matter and repetition is no allowed. However, if roles are defined in the committee like president or vice president, permutations must be used to count the committee.

Notes

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<tr><th>Guided Example 2</th><th>Practice</th></tr>
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A Congressional committee consists of 8 men and 10 women. A subcommittee of 4 people is set up at random. What is the probability that it will consist of all men?

Solution Let's start by determining how many subcommittees of four that may be selected from 18 people. Since this is a committee with no assigned roles, combinations give us

$$C(18,4)=\frac{18!}{(18-4)!4!}=3060$$
This gives us the number of outcomes in the sample space where the sample space is all subcommittees of 4 selected from 18 members.

The event we are interested in is all subcommittee members are males. Since there are 8 men in the committee, the number of 4-person male subcommittees is

$$C(8,4)=\frac{8!}{(8-4)!4!}=70$$
The likelihood of an all-male committee is

$$P(\text{all male subcommittee})=\frac{70}{3060}\approx 0.0229$$
or about 2.29%.
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A Congressional committee consists of 12 men and 6 women. A subcommittee of 4 people is set up at random. What is the probability that it will consist of all women?
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Question 3: How do you find the probability of winning a lottery?

Key Terms

Lotteries are contests in which numbers are randomly selected from a group. If the numbers a player chooses matches the numbers chosen by the organization conducting the lottery, the player wins a prize. Lesser prizes ma be awarded for matching fewer numbers.

In "The Pick" conducted by the state of Arizona, the state randomly picks six numbers from the numbers 1 through 44. If the numbers the player picks match the states numbers, the player wins the jackpot. Smaller prizes are awarded for matching five, four or three numbers. Since the order in which numbers are picked are irrelevant, combinations are used to find the number of ways to pick numbers.

Summary

Notes

Guided Example 3	Practice
A state lottery game requires that you pick 6 different numbers from 1 to 44. What is the probability of picking exactly 3 of the 6 numbers correctly? **Solution** To determine the number of ways the state can pick numbers, we find the combination of six numbers selected from 44, $$C(44,6)=\frac{44!}{(44-6)!6!}=7,059,052$$ To pick exactly three numbers correctly, we need three winning numbers and three losing numbers. The number of ways to do this is $$C(6,3)\cdot C(38,3)=20\cdot 8436=168,720$$ The likelihood of matching three of the winning numbers is $$P(\text{matching 3 numbers})=\frac{168,720}{7,059,052}\approx 0.0239$$ or about 2.39%.	A state lottery game requires that you pick 6 different numbers from 1 to 52. What is the probability of picking exactly 4 of the 6 numbers correctly?

Question 4: How do you find the likelihood of detecting a defective product?

Key Terms

Summary

When checking inventory for defective products, a manufacturer is not interested in how the defective units are ordered. They are interested in how many defective items are in a sample. For this reason, combinations are used to count the number of ways defective items may be picked from a sample.

Notes

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<tr><th><u>Guided Example 4</u></th><th><u>Practice</u></th></tr>
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A shipment of 20 cars contains 3 defective cars. Find the probability that a sample of size 2, drawn from the 20, will not contain exactly one defective car.

Solution Start by determining how many ways there are to select a sample of 2 cars from 20 cars. Since order does not matter and repetition is not allowed, combinations are used to calculate the number of ways to select a sample of 2 from 20 cars,

$$C(20,2)=\frac{20!}{(20-2)!2!}=190$$

For the sample to contain exactly one defective car, a car must be selected from the defective cars and another from the non-defective cars. The number of ways this can be done is

$$\underbrace{C(3,1)}_{\text{defective}}\cdot\underbrace{C(17,1)}_{\text{non-defective}}=3\cdot 17=51$$

By dividing the number of ways to pick exactly one defective by the number of ways to select a sample of 2 from 20, we get the likelihood of getting exactly one defective car,

$$P(\text{exactly one defective car})=\frac{51}{190}\approx 0.268$$

or approximately 26.8%.
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A bag of 40 Cadbury Easter Eggs contains 2 defective eggs. Find the probability that a sample of size 3, drawn from the 40, will contain exactly one defective egg.

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