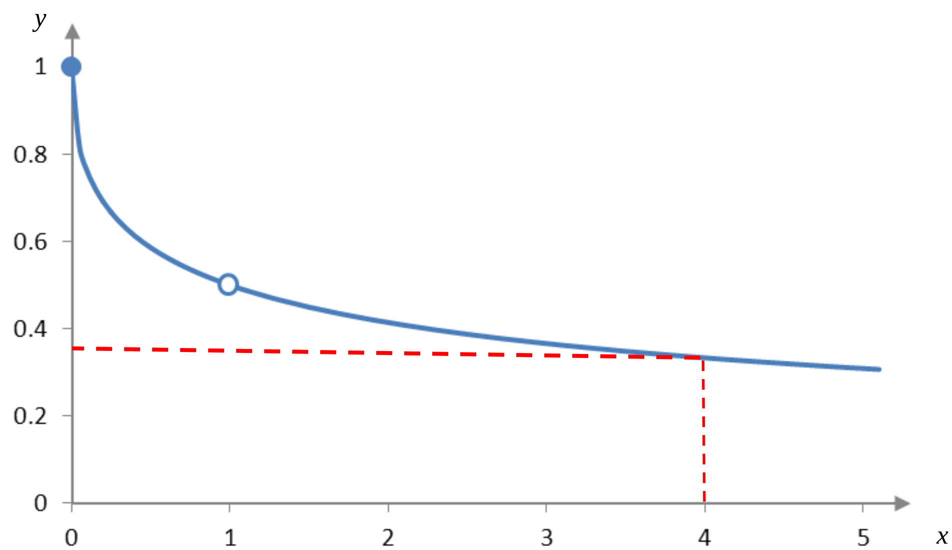
Evaluate each of the limits below.

a. $\lim_{x \to 4} \frac{\sqrt{x}-1}{x-1}$

Solution Start by substituting the value $x = 4$ into the function. This gives us

$$\lim_{x \to 4} \frac{\sqrt{x}-1}{x-1} = \frac{\sqrt{4}-1}{4-1} = \frac{1}{3}$$

We can check this limit by examining the function's graph.



As x approaches 4 from either side, the corresponding y value gets closer and closer to about $\frac{1}{3}$.

b. $\lim_{x \to 1} \frac{\sqrt{x}-1}{x-1}$

Solution If we try to substitute $x = 1$ into the fraction, the numerator and denominator are both zero. This suggests that there might be some algebra we can do to simplify the fraction. Once this simplification is done, we can substitute $x = 1$ into the simplified expression.

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}=\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}$$

Multiply the numerator and denominator by the conjugate of the numerator, $\sqrt{x}+1$

$$=\lim_{x\to 1}\frac{x-1}{(x-1)\left(\sqrt{x}+1\right)}$$

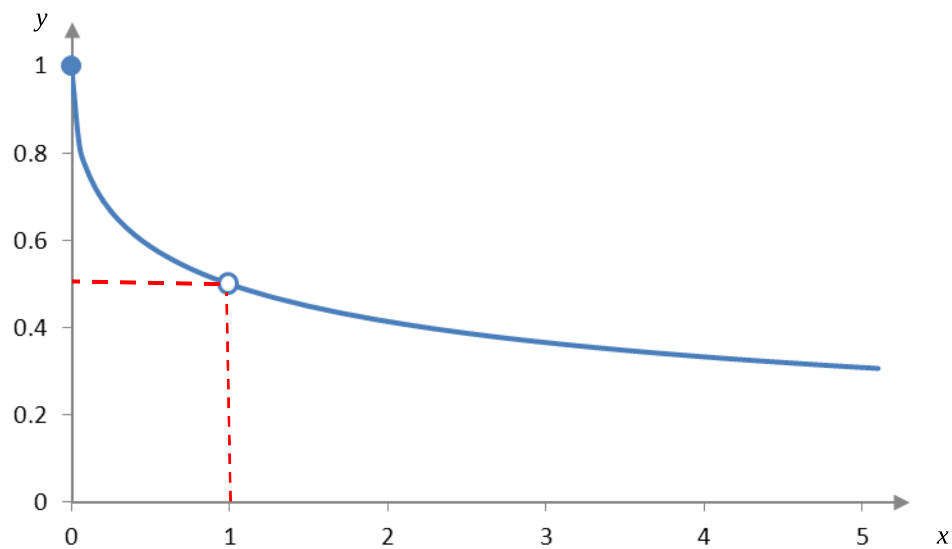
Foil the numerators,
$\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)=x+\cancel{\sqrt{x}}-\cancel{\sqrt{x}}-1$

$$=\lim_{x\to 1}\frac{1}{\sqrt{x}+1}$$

Reduce the fraction

$$=\frac{1}{2}$$

Substitute $x = 1$



Viewing the graph, as x approaches 1 the corresponding y values approach $\frac{1}{2}$.