Find the derivative of $y = \dfrac{2\ln(x+3)}{x^2}$.

To find this derivative we'll need to use the quotient rule with

$$u = 2\ln(x+3)$$
$$v = x^2$$

The derivative of v, $v' = 2x$, is easy. The harder part is the derivative of u. This will require a chain rule with the outside function being $2\ln(x)$ and the inside function being $x+3$. Applying the chain rule gives

$$u' = \tfrac{2}{x+3} \cdot 1$$

Now that we have the derivative of u and v, we can apply the quotient rule:

$$\frac{vu' - uv'}{v^2} = \frac{x^2 \cdot \frac{2}{x+3} - 2\ln(x+3)\cdot 2x}{\left(x^2\right)^2}$$

Now let's try a little simplifying:

$$\frac{x^2 \cdot \frac{2}{x+3} - 2\ln(x+3)\cdot 2x}{\left(x^2\right)^2} = \frac{x^2 \cdot \frac{2}{x+3}}{x^4} - \frac{4x\ln(x+3)}{x^4}$$
$$= \frac{2}{x^2(x+3)} - \frac{4\ln(x+3)}{x^3}$$

In the first line I have divided both terms on top by x^4. In the second line I reduced each of the resulting fractions.