Many antiderivatives can be taken using the Power Rule for Antiderivatives,

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C$$

This rule is so powerful because the values of n can be almost anything. I say anything because there is one case that makes no sense. Suppose $n = -1$. Applying the Power Rule for Antiderivatives gives

$$\int x^{-1} \, dx = \frac{1}{-1+1} x^{-1+1} + C$$

The fraction on the right side has a zero in the denominator. The Power Rule for Antiderivatives does not apply in this case. However, you'll recall that $\frac{d}{dx}[\ln(x)] = \frac{1}{x}$. Since $\frac{1}{x}$ is the same as x^{-1},

$$\frac{d}{dx}[\ln(x)] = x^{-1}$$

Reversing this process gives us the antiderivatives

$$\int x^{-1} \, dx = \ln|x| + C$$

The absolute values are needed to insure the domains of each side match. In business calculus, the inputs to our functions are generally non-negative so we can safely treat the absolute value as if they are parentheses. So in general we have the following for the Power Rule for Antiderivatives:

$$\int x^n dx = \begin{cases} \dfrac{1}{n+1} x^{n+1} + C & if\ n \neq -1 \\ \ln(x) + C & if\ n = -1 \end{cases}$$

The rule is easy to apply for basic antiderivatives. It becomes more complicated when the integrand (the expression you are taking the antiderivatives of) is not a power function, but can be rewritten as a power function.

Example 1 Find the Antiderivative

Evaluate $\int 5x(x^2 - 8)\, dx$.

Solution To be able to apply the Power Rule for Antiderivatives, carry out the multiplication,

$$\int 5x(x^2 - 8)\, d = \int (5x^3 - 40x)\, dx$$

Now we can apply the Power Rule to each term,

$$\begin{aligned}\int 5x(x^2 - 8)\, dx &= \int (5x^3 - 40x)\, dx \\ &= 5 \cdot \frac{1}{4} x^4 - 40 \cdot \frac{1}{2} x^2 + C \\ &= \frac{5}{4} x^2 - 20x^2 + C\end{aligned}$$

By carrying out the multiplication, we get a difference of power functions. The antiderivatives are easy to carry out once this is done.

Example 2 Find the Antiderivative

Evaluate $\int \left(\frac{\pi^3}{y^3} + \frac{1}{\sqrt{y}}\right)\, dy$.

Solution As with the last example, the key is to rewrite the integrand with power functions:

$$\begin{aligned}\int \left(\frac{\pi^3}{y^3} + \frac{1}{\sqrt{y}}\right)\, dy &= \int \left(\pi^3 y^{-3} + y^{-\frac{1}{2}}\right)\, dy \\ &= \pi^3 \cdot \frac{1}{-2} y^{-2} + \frac{1}{\frac{1}{2}} y^{\frac{1}{2}} + C \\ &= -\frac{\pi^3}{2} y^{-2} + 2y^{\frac{1}{2}} + C\end{aligned}$$

If desired, the power functions can also be rewritten to give $-\frac{\pi^3}{2y^2} + 2\sqrt{y} + C$.

Example 3 Find the Antiderivative

Evaluate $\int \frac{x^4+2x^2-x}{3x^2}\,dx$.

Solution Rewrite the integrand with a power function and apply the Power Rule for Antiderivatives,

$$\begin{aligned}\int \frac{x^4+2x^2-x}{3x^2}\,dx &= \int \left(\frac{1}{3}x^2+\frac{2}{3}-\frac{1}{3}\frac{1}{x}\right)dx \\ &= \frac{1}{3}\cdot\frac{1}{3}x^3+\frac{2}{3}x-\frac{1}{3}\ln(x)+C \\ &= \frac{1}{9}x^3+\frac{2}{3}x-\frac{1}{3}\ln(x)+C\end{aligned}$$

Example 4 Find the Antiderivative

Evaluate $\int (x+2)^2\,dx$.

Solution Carry out the multiplication to change the integrand to a trinomial,

$$\begin{aligned}\int (x+2)^2\,dx &= \int (x^2+4x+4)\,dx \\ &= \frac{1}{3}x^3+4\cdot\frac{1}{2}x^2+4x+C\end{aligned}$$