Find the derivative: $\frac{d}{dx}\left[\left(x^2+4x\right)\left(5x+6\right)^4\right]$

To take the derivative, we need to write the function as a product with

$$u = x^2 + 4x \qquad v = \left(5x+6\right)^4$$

The product rule requires the derivative of each of these factors. The derivative of the first factor is

$$u' = 2x + 4$$

The second factor is a bit more complicated and requires the use of the chain rule. Rewrite the factor as a composition $f\left(g\left(x\right)\right)$ with

$$g\left(x\right) = 5x + 6$$

$$f\left(x\right) = x^4$$

The derivative of these functions are

$$g\left(x\right) = 5x + 6 \quad \rightarrow \quad g'\left(x\right) = 5$$

$$f\left(x\right) = x^4 \quad \rightarrow \quad f'\left(x\right) = 4x^3$$

Apply the chain rule to obtain

$$v' = \underbrace{4\left(5x+6\right)^3}_{f'(g(x))} \cdot \underbrace{5}_{g'(x)}$$

Now put this derivative into the product rule with the derivatives above,

$$\frac{d}{dx}\left[\left(x^2+4x\right)\left(5x+6\right)^4\right] = \underbrace{\left(5x+6\right)^4}_{v} \cdot \underbrace{\left(2x+4\right)}_{u'} + \underbrace{\left(x^2+4x\right)}_{u} \cdot \underbrace{20\left(5x+6\right)^3}_{v'}$$

to give the derivative we are looking for. This can be simplified by factoring $\left(5x+6\right)^3$ from each term, but we'll stop here since the derivative has been completed.