Section 4.1 Linear Equations

- How do you solve a linear equation?
- How can intercepts be used to graph linear equations?

How do you solve a linear equation?

Key Terms

Linear Growth Linear Equation in Two Variables

Summary

This chapter is about growth, and how to model growth with equations. We will be investigating linear growth, quadratic growth, and exponential growth.

Typically, we measure a quantity with respect to time, so **linear growth refers to a quantity that changes at a constant rate for each unit of time**. For example, let's say you graduate college and get a job that pays $1500 a week (after taxes, benefits, etc). You could model this scenario with the equation $y = 1500x$ where y is the amount of money you earn after x weeks of working. Notice the growth rate is 1500 per week, and it stays the same each week.

The equation above is an example of a **linear equation in two variables: an equation that can be written in the form $Ax + By = C$, where *A, B,* and *C* are real numbers (*A* and *B* can't both be zero)**. Later we will investigate quadratic equations that are characterized by having an x^2 term, and then exponential equations that are characterized by having a b^x term.

When solving linear equations, keep these two important algebraic properties in mind:

1. You can add or subtract a quantity on both sides of an equation.
2. You can multiply or divide by a nonzero quantity on both sides of an equation.

Notes

Guided Example 1	Practice
Recall our job scenario above and suppose that that you had saved \$3000 in your checking account before you started the job, and your paycheck is directly deposited each week. You could model this scenario with the equation $y = 1500x + 3000$ where y is the amount of money in your checking account after x weeks of work. Use this equation to figure out how many weeks of work it would take you to have \$12,000 in your checking account. **Solution** Since y represents the amount of money in the checking account, we substitute 12,000 for y and then solve for x. $12000 = 1500x + 3000$ Substitute 12000 for y. $900 = 1500x$ Subtract 3000 from both sides. $6 = x$ Divide both sides by 1500. So it will take **6 weeks** of work to have \$12,000 in your checking account.	Suppose you run a small carpentry business, and you currently have 15 cabinets in your inventory, and can produce 5 new cabinets per week. You could model this scenario with the equation $y = 5x + 15$, where y is the number of cabinets you have available for installation in x weeks. If the local high school needs 45 cabinets for their new art building, how many weeks would it take to fill that order?

Guided Example 2	Practice
Solve $8x+4y=12$ for y. **Solution** Isolate y by carrying out the steps below. $8x+4y=12$ $4y=-8x+12$ Subtract $8x$ from both sides $y=\frac{-8x+12}{4}$ Divide both sides by 4 $y=-2x+3$ Divide each term by 4 Notice our result is also a linear equation in two variables, and it is equivalent to the equation we started with. By solving for y we have simply rearranged the terms in the equation.	Solve $3a-7b=42$ for a.

How can intercepts be used to graph linear equations?

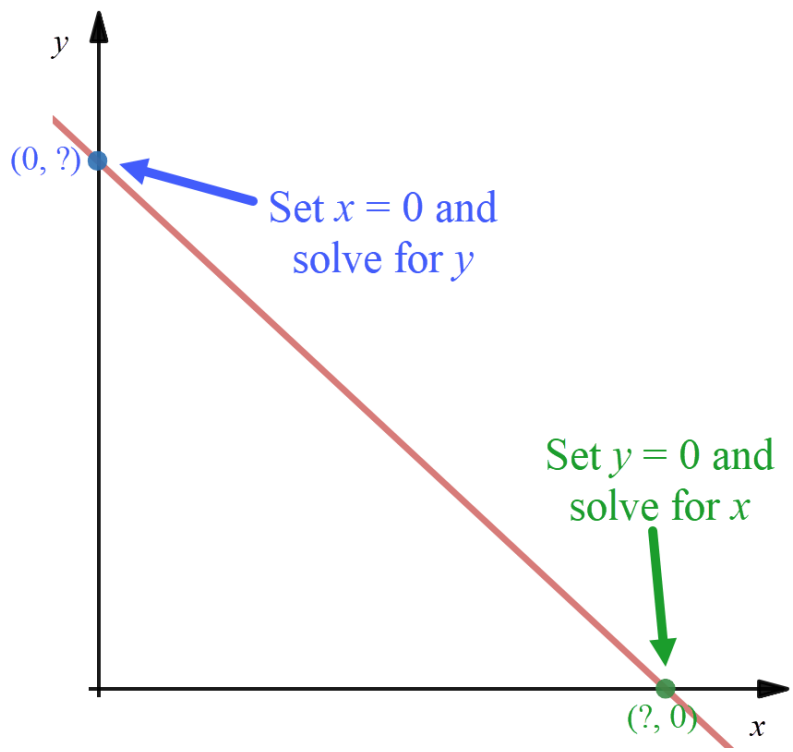
Key Terms

x-intercept *y*-intercept

Summary

The graph of a linear equation in two variables is a straight line, and the points where a line crosses the *x* and *y* axes are called intercepts. The ***x*-intercept of a line is the point where the line crosses the *x*-axis**. You can **solve for an *x*-intercept by setting *y* equal to zero and solving for *x***. This is because any point on the *x*-axis must have a *y*-coordinate of zero.

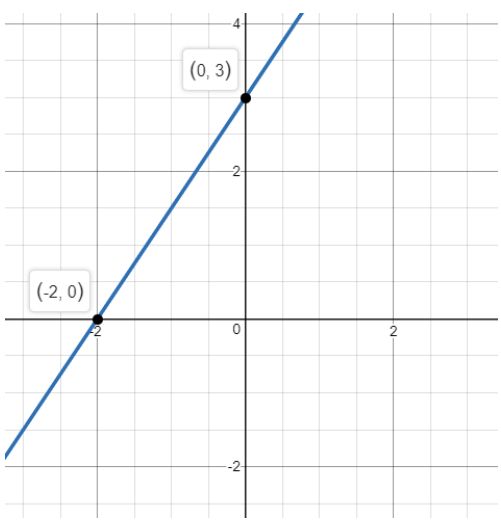
Similarly, the ***y*-intercept of a line is the point where the line crosses the *y*-axis**. You can s**olve for a *y*-intercept by setting the *x*-coordinate equal to zero and solving for *y***.



All straight lines have both an *x*-intercept and *y*-intercept, except horizontal and vertical lines which only have one or the other.

You need at least two points in order to graph a line, so once you have the intercepts, you can easily plot these and sketch the line that passes between them.

Notes

Guided Example 3	Practice
Find the x and y intercepts of the line given by the equation $-3x+2y=6$ and use them to sketch a graph of the line. **Solution** x-intercept: Set y to zero and solve for x. $-3x+2(0)=6$ Set y equal to 0 $-3x=6$ Simplify $x=-2$ Divide both sides by -3 So, the x-intercept is (-2, 0). y-intercept: Set x to zero and solve for y. $-3(0)+2y=6$ Set x equal to 0 $2y=6$ Simplify $y=3$ Divide both sides by 2 So, the y-intercept is (0, 3). Next, we will plot these points, and sketch the line that passes between them.  	Find the x and y intercepts of the line given by the equation $6x-5y=-30$ and use them to sketch a graph of the line.

Guided Example 4

Suppose you are hired to mow the lawn at a local park. If the park has 5 acres of grass, and it takes you an hour to mow half an acre on a riding lawn mower, you can model this scenario with the equation $y = -\frac{1}{2}x + 5$ where y is the number of acres left to mow after x hours. Find the x and y intercepts of this equation, interpret their meaning, and use them to sketch a graph of the linear model.

Solution

x-intercept: Set y to zero and solve for x.

$0 = -\frac{1}{2}x + 5$	Set y equal to 0
$\frac{1}{2}x = 5$	Add $\frac{1}{2}x$ to both sides
$x = 10$	Multiply both sides by 2

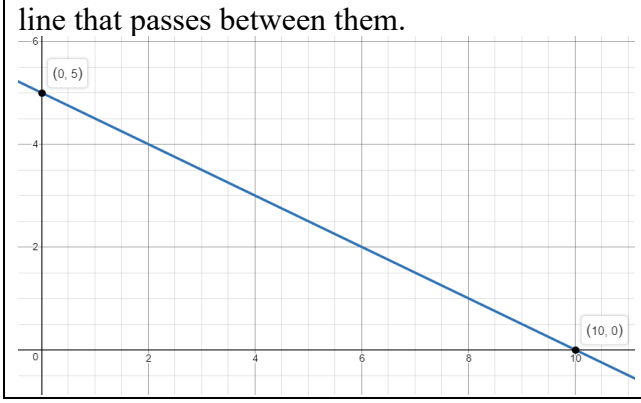
So, the x-intercept is (10, 0). In context this means after 10 hours of mowing, you have 0 acres left to mow, meaning it takes you 10 hours to complete the job.

y-intercept: Set x to zero and solve for y.

$$y = -\frac{1}{2}(0) + 5 = 5$$

So, the y-intercept is (0, 5). In context this means after 0 hours of mowing, you still have 5 acres left to mow, meaning at the start of the job there are 5 acres to mow.

Next, we will plot these points, and sketch the line that passes between them.



Practice

Suppose you need to drain your swimming pool, and you use a pump that can remove 500 gallons per hour. You can model this scenario with the equation $y = -500x + 2000$ where y is the number of gallons left in the pool after x hours of pumping. Find the x and y intercepts of this equation, interpret their meaning, and use them to sketch a graph of the linear model.